Parametric Structure-Preserving Model Order Reduction

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***Abstract*—Analysis and verification environments for next-generation nano-scale RFIC designs must be able to cope with increasing design complexity and to account for new effects, such as process variations and Electromagnetic (EM) couplings. Designed-in passives, substrate, interconnect and devices can no longer be treated in isolation as the interactions between them are becoming more relevant in the behavior of the complete system. At the same time variations in process parameters lead to small changes in the device characteristics that may directly affect system performance. These two effects, however, can not be treated separately as the process variations that modify the physical parameters of the devices also affect those same EM couplings. Accurately capturing the effects of process variations as well as the relevant EM coupling effects requires detailed models that become very expensive to simulate. Reduction techniques able to handle parametric descriptions of linear systems are necessary in order to obtain better simulation performance. In this work Model Order Reduction techniques able to handle parametric system descriptions are presented. Such techniques are based on Structure-Preserving formulations that are able to exploit the hierarchical system representation of designed-in blocks, substrate and interconnect, in order to obtain more efficient simulation models.**

I. Introduction

New coupling and loss mechanisms, including EM field coupling and substrate noise as well as process-induced variability, are becoming too strong and too relevant to be neglected, whereas more traditional coupling and loss mechanisms are more difficult to describe given the wide frequency range involved and the greater variety of structures to be modeled. The performance of each device in the circuit is strongly affected by the environment surrounding it. In other words, the response of each circuit part depends not only on its own physical and electrical characteristics, but to a great extent also on its positioning in the IC, i.e. on the devices to which it is directly connected to or coupled with. The high level of integration available in current RFIC designs leads to proximity effects between the devices, which induce EM interactions, that can lead to different behaviors of the affected parts. In any manufacturing process there is always a certain degree of uncertainty involved given our limited control over the environment. For the most part this uncertainty was previously ignored when analyzing or simulating complete systems, or assumed to be accounted for in the individual device models. However, as we step towards the nano-scale and higher frequency eras, such environmental, geometrical and electromagnetic fluctuations become more significant. Nowadays, parameter variability can no longer be disregarded, and its effect must be accounted for in early design stages so that unwanted consequences can be minimized. This leads to parametric descriptions of systems, including the effects of manufacturing variability, which further increases the complexity of such models. Reducing this complexity is paramount for efficient simulation and verification. However, the resulting reduced models must retain the ability to capture the effects of small fluctuations, in order to accurately predict behavior and optimize designs. This is the realm of *Parametric Model Order Reduction* (pMOR). Furthermore, these parametric fluctuations of the physical characteristics of the devices can affect not only the performance of such device, but the coupling between devices. For this reason the parametric models of the individual blocks of a system can no longer be simulated in isolation but must be treated as one entity and verified together. Such reduction must take advantage of the hierarchical description of those systems namely to account for designed-in elements as well as interconnect effects. To this end, structure-preserving techniques must be used which not only retain structural properties of the individual systems but also its connections and couplings.

The goal of this paper is therefore to discuss and present techniques for model order reduction of interconnect, substrate or designed-in passives, taking into account their dependence on relevant process or fabrication parameters and their coupling and connections. The paper is structured as follows: in Section II an overview of several existing pMOR techniques will be discussed. In Section III an introduction to two-level hierarchy MOR will be done, and an extension to improve the reduction will be presented. In Section IV the proposed methodology for combining the parametric techniques with the hierarchical reduction will be proposed. To illustrate the procedure, its pros and cons, in Section V some reduction results will be presented for several real-life structures. Finally conclusions will be drawn in Section VI.

II. Parametric Model Order Reduction

Actual fabrication of physical devices is susceptible to the variation of technological and geometrical parameters due to deliberate adjustment of the process or from random deviations inherent to the manufacturing procedures. This variability leads to a dependence of the extracted circuit elements on several parameters, of electrical or geometrical origin. This dependence results in a parametric state-space system representation, which in descriptor form can be written as

$$\begin{aligned} C(\lambda)\dot{x}(t,\lambda)(\lambda) + G(\lambda)x(t,\lambda) &= Bu(t) \\ y(t,\lambda) &= Lx(t,\lambda) \end{aligned} \tag{1}$$

where $C, G \in \mathbb{R}^{n\times n}$ are respectively the capacitance and conductance matrices, $B \in \mathbb{R}^{n\times m}$ is the matrix that relates the input vector $u \in \mathbb{R}^m$ to the inner states $x \in \mathbb{R}^n$ and $L \in \mathbb{R}^{n\times p}$ is the matrix that links those inner states to the outputs $y \in \mathbb{R}^p$. The elements of the matrices C and G, as well as the states of the system x, depend on a set of P parameters $\lambda = [\lambda_1, \lambda_2, \ldots, \lambda_P]$ which model the effects of the mentioned uncertainty. Usually the system is formulated so that the matrices related to the inputs and outputs (B and L) do not depend on the parameters. This time-domain descriptor yields a parametric dependent frequency response modeled via the transfer function

$$H(s,\lambda) = L(sC(\lambda) + G(\lambda))^{-1}B \tag{2}$$

for which we seek to generate a reduced order approximation, able to accurately capture the input-output behavior of the system for any point in the multidimensional frequency-parameter space.

$$\hat{H}(s,\lambda) = \hat{L}(s\hat{C}(\lambda) + \hat{G}(\lambda))^{-1}\hat{B} \tag{3}$$

In general, one attempts to generate a *Reduced Order Model* (ROM) whose structure is as much similar to the original as possible, i.e. exhibiting a similar parametric dependence and retaining as much of the original structure as possible. Many techniques have been proposed to tackle this problem and in the following we review some of the most commonly used.

A. Multi-Dimensional Moment Matching

These techniques appear as extensions to nominal moment-matching techniques [1], [2], [3]. Moment matching algorithms have gained a well deserved reputation in nominal MOR due to their simplicity and efficiency. The extensions of these techniques to the parametric case are usually based in the implicit or explicit matching of the moments of the parametric transfer function (2). This type of algorithms assumes small fluctuations of the parameters, so that an affine model based on the Taylor Series expansion can be used for approximating the behavior of the conductance and capacitance, $G(\lambda)$ and $C(\lambda)$, expressed as a function of the parameters. The Taylor series can be extended up to the desired (or required) order, including cross derivatives, for the sake of accuracy. Some schemes, denoted as Multi-Parameter Moment Matching use this idea to match, via different approaches, the multi-parameter moments of the parametric transfer function (2) (for details see [4], [5], [6]). However these methods usually suffer of oversize when the number of moments to match is high.

A slightly different approach, that provides more compact ROMs, is presented in [7], which relies on the computation of several subspaces, built separately for each dimension, i.e. the frequency s and the parameter set λ. Given a parametric system (1), the first step of the algorithm is to obtain the k_s block moments of the transfer function with respect to the frequency when the parameters take their nominal value (for example, via [1]). This block moments will be denoted as Q_s. The next step is to obtain the subspace which matches the k_{λ_i} block moments of x with respect to each of the parameter λ_i, and will be denoted by Q_{λ_i}. Once all the subspaces have been computed, an orthonormal basis can be obtained so that its columns spans the union of all previously computed subspaces. Applying the resulting matrix in a projection scheme ensures that the parametric ROM matches k_s moments of the original system with respect to the frequency, and k_{λ_i} moments with respect to the parameter λ_i.

B. Variational PMTBR

A novel approach was recently proposed that extends the PMTBR [8] algorithm to include variability [9]. This approach is based on the statistical interpretation of the algorithm (see [8] for details) and enhances its applicability. In this interpretation, the approximated Gramian is seen as a covariance matrix for a Gaussian variable, $x(0)$, obtained by exciting the underlying system description with white noise. Rewriting the Gramian as

$$X_\lambda = \int_{S_\lambda}\int_{-\infty}^{\infty} (j\omega C_\lambda + G_\lambda)^{-1}BB^T(j\omega C_\lambda + G_\lambda)^{-H}p(\lambda)dwd\lambda \tag{4}$$

where $p(\lambda)$ is the *Probability Density Function* (PDF) of λ in the parameter space, S_λ. Just as in the original PMTBR algorithm, a quadrature rule can be applied in the parameter plus frequency space to approximate the Gramian via numerical computation (see [9] for details). The accuracy of the resulting ROM does not depend on the accuracy of the approximation of the integral, but on the projection subspace. After the quadrature is performed in the overall variational subspace, the deterministic procedure is followed and the most relevant vectors are selected via *Singular Value Decomposition* (SVD) in order to build a projection matrix meant to be used as a congruence transformation on the parametric system matrices (1). As in the deterministic case, an error analysis and control can be included, via the eigenvalues of the SVD, but in this variational case, only an expected error bound can be given The complexity and computational cost is generally the same as that of the deterministic PMTBR plus the previous quadrature operations, and, it has been shown that the size of the reduced model is less sensitive to the number of parameters in the description, or how this parameter dependence is modeled.

III. Block Hierarchical Model Order Reduction

A. Structure Preservation

As pointed out, individual blocks inside an RFIC can no longer be treated in isolation, and for this reason the complete system must be treated as an entity. Considering the linear component blocks including designed-in passives, interconnect, etc, the system description has an interesting structure, where the diagonal blocks correspond to the individual block matrices, whereas the off-diagonal blocks correspond to the static interconnections (in the G matrix) and dynamic couplings (C matrix). Standard model order reductions techniques can be applied to this joint, global system and while the resulting reduced model will usually be able to accurately capture the input-output behavior of the complete set of blocks, this approach leads to full reduced matrices. Furthermore, the original two-level hierarchy with interconnections and couplings can no longer be recovered.

An alternative approach is to perform the reduction of the individual models in a hierarchical fashion, i.e to reduce each model independently without taking into account the rest of the models or the environment. Hence every model is reduced separately and thus the hierarchy and structure of the global system is maintained. However, to apply MOR to each model means to capture its individual behavior, not the global one. This can be inefficient as too much effort may be spent capturing some local behavior that is not relevant for the global response (maybe filtered by another model). Furthermore certain aspects of the global response might be missed as it is not clear at the component level how relevant they are. To avoid these problems, one can reduce each component block separately but oriented to capture the global input-output response. This approach will provide us with more control in the reduction stage while also preserving the structure of the interconnections. The transfer function to match is the global one, so the most relevant behavior for the complete RF system is captured. What is more, only the global inputs and outputs of the complete RF block are relevant, so the inefficiencies caused by the large number of ports of the individual component blocks is avoided.

Some recent methods have advocated this approach. In [10] a control theoretic viewpoint of reduction of interconnected systems was presented, but it has the disadvantage that it is unable to treat capacitive couplings. The *Block Structure Preserving* (BSP) technique was first presented in [11] and later generalized in [12].

$$G=\begin{bmatrix} G_{11} & \dots & G_{1N_b} \\ \vdots & \ddots & \vdots \\ G_{N_b1} & \dots & G_{N_bN_b} \end{bmatrix} \quad C=\begin{bmatrix} C_{11} & \dots & C_{1N_b} \\ \vdots & \ddots & \vdots \\ C_{N_b1} & \dots & C_{N_bN_b} \end{bmatrix}$$
$$B=\begin{bmatrix} {B_1}^T & \dots & {B_{N_b}}^T \end{bmatrix}^T \qquad L=\begin{bmatrix} L_1 & \dots & L_{N_b} \end{bmatrix} \tag{5}$$

The main idea was to retain the system block structure, i.e. the two-level hierarchy, after reduction via projection, allowing for a more efficient reduction and the maintenance of certain system properties, such as the degree of sparsity,

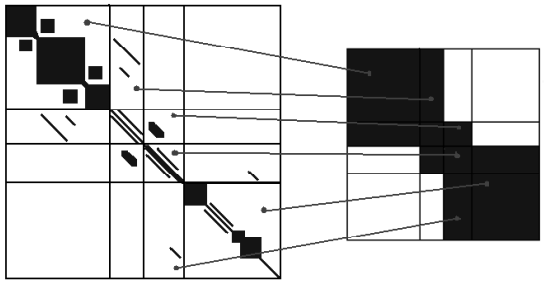

Fig. 1. Illustration of block hierarchy in the system matrix and effect of reduction using BSP.

and the block hierarchical structure. The procedure relies on expanding the projector of the global system (obtained via any classical MOR projection technique) into a block diagonal matrix, with block sizes equal to the sizes of its N_b individual component blocks (5). A basis that spans a suitable subspace for reduction via projection is then computed (for example a Krylov subspace). The projector built from that basis can be split and restructured into a block diagonal one so that the 2-level structure is preserved under congruence transformation.

$$\begin{bmatrix} V_1 \\ \vdots \\ V_{N_b} \end{bmatrix} \equiv colsp\left[Kr\left\{A,R,q\right\}\right] \rightarrow \begin{bmatrix} V_1 & & \\ & \ddots & \\ & & V_{N_b} \end{bmatrix} = \breve{V} \tag{6}$$

where $Kr\left\{A,R,q\right\}$ is the q column Krylov subspace of the complete system ($A=G^{-1}C$ and $R=G^{-1}B$). The block-wise congruence transformation is (see Figure 1)

$$\begin{array}{ll} \hat{G}_{ij}=V_i^T G_{ij} V_j & \hat{B}_i=V_i^T B_i \\ \hat{C}_{ij}=V_i^T C_{ij} V_j & \hat{L}_j=L_j V_j \end{array} \tag{7}$$

It should be noticed that the above projection matrix $\breve{V}$ has N_b (number of blocks) times more columns than the original projector . This leads to an N_b times larger reduced system. On the other hand, this technique maintains the block structure of the original system and gives us some flexibility when choosing the size of the reduced model depending on the block layout and relevance. The reduced system will be able to match up to N_b times q block moments of the original complete transfer function (see [12] for details) under the best conditions (i.e. with very weak entries in the off-diagonal blocks). Under the worst conditions, only q block moments are matched, i.e. the same number than in the *flat* reduction. This technique is applicable to the global system, composed of the individual blocks and their connections (including both resistive as well as capacitive or inductive couplings between the blocks). The BSP technique therefore preserves the block structure of the system. However, the inner structure of the blocks themselves is lost since the procedure turns any non-empty block in the original system into a full block, but it is still possible to identify the blocks and relate them to the original device or interaction block. Nevertheless, if any block is empty in the global system matrix, it remains empty after reduction, increasing the sparsity.

B. PMTBR in Block Structure MOR

Any projection-based MOR procedure can be extended in the BSP manner to maintain the hierarchical structure of a system. In the case of the PMTBR algorithm, additional characteristics of the procedure can be further taken advantageous of in the current framework. The PMTBR algorithm links the rational projection methods with the *Truncated Balanced Realizations* (TBR) framework [13]. The procedure is based on the estimation, via a quadrature rule, of the frequency-based integral expression for the controllability Gramian, (4),

$$\bar{X} = \sum_k z_k z_k^H = ZZ^H \tag{8}$$

where $Z = [z_1\ z_2\ \ldots]$ and $z_i = (jw_iC + G)^{-1}B$. In [8] it was shown that if the quadrature scheme (8) is accurate enough, then the estimated Gramian $\bar{X}$ converges to the original one X, which implies that the dominant eigenspace of $\bar{X}$ converges to the dominant eigenspace of X. If the system has some internal structure, then the matrix Z that is computed from the vector samples of the global system can be split into blocks. The estimated Gramian can be written block-wise as

$$\begin{bmatrix} Z_1 \\ \vdots \\ Z_{N_b} \end{bmatrix} \rightarrow ZZ^H = \begin{bmatrix} Z_1Z_1^H & \ldots & Z_1Z_{N_b}^H \\ \vdots & \ddots & \vdots \\ Z_{N_b}Z_1^H & \ldots & Z_{N_b}Z_{N_b}^H \end{bmatrix} = \bar{X} \tag{9}$$

But if we expand the matrix Z into diagonal blocks

$$\breve{Z} = \begin{bmatrix} Z_1 & & \\ & \ddots & \\ & & Z_{N_b} \end{bmatrix} \rightarrow \breve{Z}\breve{Z}^H = \begin{bmatrix} Z_1Z_1^H & & \\ & \ddots & \\ & & Z_{N_b}Z_{N_b}^H \end{bmatrix} = \breve{X}. \tag{10}$$

From (9) it can be seen that $Z_iZ_i^H = \bar{X}_{ii}$, i.e. the matrix $\breve{X} = \breve{Z}\breve{Z}^H$ is a block diagonal matrix whose entries are the block diagonal entries of the matrix $\bar{X}$. Under a good quadrature scheme, the matrix $\bar{X}$ converges to the original X, and therefore $\breve{X}$ will converge to the block diagonals of X. This means that the dominant eigenspace of $\breve{X}$ converges to the dominant eigenspace of the block diagonals of X. We can then apply an SVD to each block of the Z matrix

$$Z_i = V_iS_iU_i \ \rightarrow\ \breve{X}_{ii} = \bar{X}_{ii} = V_iS_i^2V_i^T \tag{11}$$

where S_i is real diagonal, and V_i and U_i are unitary matrices. The dominant eigenvectors of V_i corresponding to the dominant eigenvalues of S_i can be used as a projection matrix in a congruence transformation over the system matrices for model order reduction. The elements of S_i can also be used for a priori error estimation in a way similar to how Hankel Singular Values are used in TBR procedures. Using these block projectors V_i, a structure preserving projector for the global system can be built (6) which will capture the most relevant behavior of each block (revealed by the SVD) with respect to the global response (recall that Z is composed of sample vectors of the complete system). This approach provides us with more flexibility when reducing a complete system composed of several blocks and the interactions between them, as it allows to control the reduced size of each device via an error estimation on the global response.

IV. Parametric Block Structure MOR

From the two-level hierarchical description of a system it is possible to have some extra block information that allows us to perform a more efficient MOR. But the behavior of the individual blocks that compose the system is subject to the effect of process variations, both geometrical and electrical. Such variations, as previously pointed out, also affect the interactions and couplings between these blocks. Any system-wide EM simulations must address these effects. Therefore, the variability study must be done over the complete system, and after model generation, a two-level parametric system will be obtained, with the block matrices in the block diagonals and the interactions between them in the off-diagonals. All these blocks will be functions of the relevant process and geometrical parameter. For instance, for conductivity,

$$G = \begin{bmatrix} G_{11}(\lambda_{\{11\}}) & \ldots & G_{1N_b}(\lambda_{\{1N_b\}}) \\ \vdots & \ddots & \vdots \\ G_{N_b1}(\lambda_{\{N_b1\}}) & \ldots & G_{N_bN_b}(\lambda_{\{N_bN_b\}}) \end{bmatrix} \tag{12}$$

where $\lambda_{\{ij\}}$ represents the set of parameters affecting block G_{ij}. From (12) is clear that we have a parametric system depending on $\lambda = \left[\lambda_{\{11\}} \ldots\ \lambda_{\{N_bN_b\}}\right]$. Therefore we can apply parametric MOR reduction. Note that any parameter affecting several blocks (diagonal blocks and their interactions) is treated as a single parameter (this reduces the number of parameters). However, in order to maintain the system structure, BSP techniques can be applied. This is possible as long as the selected pMOR technique is based in a projection scheme, which is the case for most of the existing procedures. The extension is very simple: obtain a suitable basis for projection from the ***complete system***, and then split and expand it into a block structure preserving projector. If the basis spans the most relevant behavior of the parametric system, then the expanded BSP projector will capture those as well. All the advantages and disadvantages mentioned in Section III hold here. But there is an extra and important advantage in the parametric case: **the BSP technique maintains the block parametric dependance**, i.e. if a block C_{ij} depends on a set of parameters $\lambda_{\{ij\}}$, then the reduced block $\hat{C}_{ij} = V_i^TC_{ij}V_j$ will depend on the same parameter set and no other.

On the other hand, as previously discussed some pMOR algorithms yield a very large ROM, and therefore their combination with BSP techniques will lead to an extremely large ROM. However, it was shown in Section II-B that the ROM size of the Variatinal PMTBR method is less sensitivity to the number of parameters. Furthermore, this method has a direct relation with PMTBR: the only difference is in the sampling scheme for obtaining the matrix whose columns spans the desired subspace, the rest of the procedure being exactly the same. Therefore, the results obtained in Section III-B are applicable to the variational case. The advantage of the control and error estimation still remains, although in this case only an expected error bound can be given. Such control is very useful when the models of a complete entity have very different sizes: if the same ROM size is applied to every

Algorithm I: Block Structure Preserving VPMTBR

Starting from a Block Structured System C, G, B, L with N_b blocks:

1: Select a quadrature rule of K points in the space $[s, \lambda]$
2: For each point compute: $z_i = (s_i C(\lambda_i) + G(\lambda_i))^{-1} B$
3: Form the matrix columns $Z = [z_1 \dots z_k]$
4: Split it into N_b blocks,

$$Z = \begin{bmatrix} Z_1 \\ \vdots \\ Z_{N_b} \end{bmatrix}$$

5: For each block Z_j obtain the SVD: $Z_j = V_j S_j U_j$
6: For each matrix V_j drop the columns whose singular values falls bellow the desired global tolerance
7: Build a Block Structure Preserving Projector from the remaining columns

$$\breve{V} = \begin{bmatrix} V_1 & & \\ & \ddots & \\ & & V_{N_b} \end{bmatrix}$$

8: Apply $\breve{V}$ in a congruence transformation on the Block Structured System C, G, B, L

block, the reduction may grow unnecessarily large. In contrast, the complexity of the proposed methodology is exactly the same as that for the non-structure-preserving techniques. The only difference is that the SVD (or orthonormalization in the moment matching approaches) must be done block-wise in order to avoid numerical errors. This can turn into an advantage, because for some blocks the number of vectors needed is lower, so less computational effort is required.

V. Results

To illustrate the proposed procedure we present results from two examples to which several pMOR techniques were applied. These included [9] denoted as VPMTBR, [7] denoted as PPTDM, and two Block Structure preserving methods: *Algorithm I*, denoted as BS VPMTBR, and block struture based on [7], denoted as BS PPTDM. The non-reduced model response will be denoted as Original or Perturbed, depending on whether a parameter variation has been applied.

A. Example 1 - Spiral

The first example system is composed of three blocks: a *Multiple Input Multiple Output* (MIMO) RC ladder of size 101, with 2 ports, a MIMO Spiral Inductor of size 4961, with 2 ports, and another RC ladder of size 101 and 2 ports. The three systems are connected in series as shown in Figure 2, so the global input is the input of the first RC and the output is the output of the second RC. The Spiral has each of its ports conected to each ladder. The system depends on five parameters, affecting different blocks. Figure 3 shows the frequency response of the self-admittance Y_{11} of the nominal system, the pertubed response of the non-reduced system, and the responses of the PMTBR-based models (the PPTDM and BS PPTDM models do not produce competitive results sizewise,

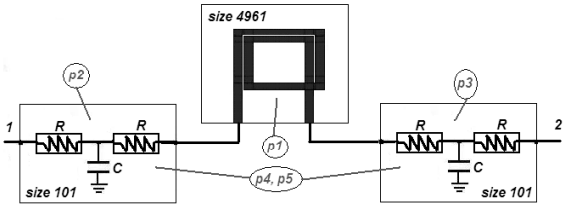

Fig. 2. Interconnection scheme for Example 1, with original sizes and parameter indication.

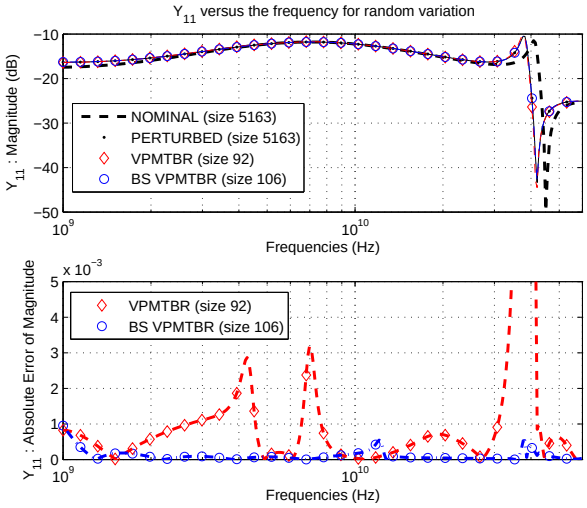

Fig. 3. (Up) Magnitude in dB of Y_{11} versus the frequency of Example 1 for the nominal, the pertubed and the parametric ROMs for a random parameter variation. (Down) Error of the Magnitude of Y_{11} for the ROMs w.r.t. the perturbed response.

TABLE I
Characteristics of the pMOR methods applied

	Example 1		Example 2	
MOR Method	Size	NNZ (G C)	Size	NNZ (G C)
NONE	5163	22545 6631	1600	4768 12588
VPMTBR	92	8464 8464	66	4356 4356
PPTDM	150	22500 22500	544	295936 295936
BS VPMTBR	106	11108 8228	96	722 5438
BS PPTDM	352	103502 42856	160	1600 17200

as seen from Table I, and therefore were omitted). Table I shows the main characteristics of the obtained ROMs. The moment matching techniques are less efficient, as the Spiral requires a high-order model. The PMTBR-based techniques obtain a better compression overall: BS VPMTBR yields a sligthy bigger ROM, but it maintains the block structure of the original system, and is able to control the size of each reduced block depending on its relevance on the global response.

B. Example 2 - Coupled Buses

This example, depicted in Figure 4, is composed of 16 blocks: 2 buses of 8 parallel lines each (each line modeled as an RC ladder of 100 segments), are on different metal layers, and cross at a square angle. The inputs and outputs are taken at the edges of each line of the first bus, so the system will have 16 ports. In this case there is no interconnection, just coupling

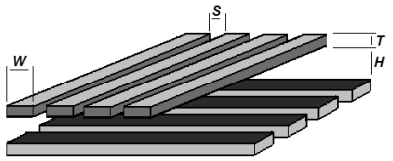

Fig. 4. Bus topology for Example 2.

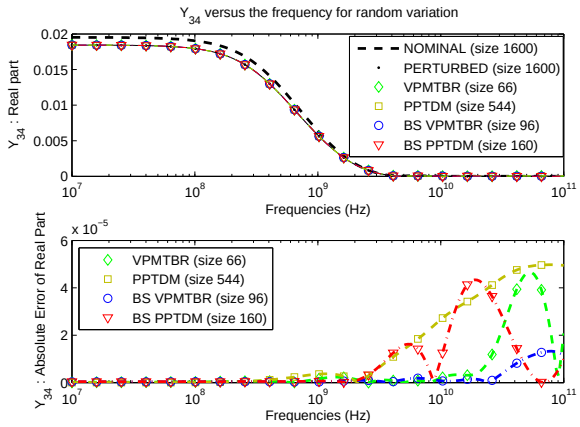

Fig. 5. (Up) Y_{34} versus the frequency for Example 2 for the nominal, pertubed and parametric ROMs with random parameter variation set. (Down) Absolute Error of the ROMs w.r.t. the perturbed response.

effects. Each line is assumed coupled to the previous and the next line of their bus, and to every line of the other bus in the crossing area. Each line has its width (W) as a parameter, which implies 16 independent parameters. The width variation affects the line model, as well as the in-bus coupling (width variation also affects the interline spacing), and the inter-bus coupling (the crossing area varies). Figure 5 shows the frequency response of the nominal system, the pertubed response of the non-reduced system, and the responses of the ROMs for VPMTBR, PPTDM, BS VPMTBR and BS PPTDM. Again, the main characteristics of the resulting ROMs are shown in Table I. The PPTDM based algorithms result in very large ROMs even for small number of moments to match (2 w.r.t. the frequency and 2 w.r.t. each parameter). For these reasons each block moment from PPTDM was truncated to 10 vectors to keep the size manageable (otherwise no reduction would be possible). While this seems to produce acceptable results, there is little control over the result. On the other hand, the PMTBR based techniques leads to more compressed ROMs, as the SVD reveals the most relevant vectors. In the case of the BS VPMTBR, the control of each block allows different reduction sizes for each bus: since the ports of the 2nd bus are not taken into account, less effort is needed to capture its behavior. In fact, the models for the 1^{st} bus are of sizes 8 to 10, while models for the 2^{nd} bus are all size 3. The ability to control reduction locally is clearly an advantage of the method.

VI. Conclusion

In this paper we have presented a block structure-preserving parametric model order reduction technique, as an extension of existing pMOR techniques in order to improve the reduction when a two-level hierarchical structure is available in the system description. This type of structure is common in coupled or interconnected systems, and can lead to simulation advantages. The methodology presented here is general as it can be used with any projection pMOR technique to mantain the two-level hierarchy and the block-parameter dependance. The presented extension of the PMTBR-based procedures into the Block Structure Preserving framework, allows more control on the reduction, provided by the inclusion of estimated error bounds on the single blocks oriented to the global response.

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