Effective Domain Partitioning With Electric and Magnetic Hooks

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This paper discusses interface conditions used in a domain partitioning method and their approximation with a reduced number of degrees of freedom called hooks. In the electrostatic and magnetostatic cases, electric or magnetic hook-connectors are, respectively, used to describe interactions. Better numerical results are obtained in the full wave regime by using both electric and magnetic hooks. This paper proposes an efficient approximation of the interface conditions by using a coarser grid on this surface. We have shown that the interface tends to become transparent for the electromagnetic (EM) field, when the number of hooks is increased and consider this convergence property as the main result of the paper. The proposed domain partitioning (DP) method was successfully applied as a particular domain decomposition (DD) technique for the EM modeling with parallel algorithms of RF-IC components. Unlike DD which is an iterative approach, the new DP approach is a direct one. The sub-domain models being independently extracted, DP is more effective and suitable for parallelization. The open problem of hooks identification is reformulated as a discrete optimization problem.

***Index Terms*—Domain decomposition (DD)/partitioning, electromagnetic (EM) analysis, electromagnetic (EM) circuit element, finite integration technique (FIT).**

I. Introduction

NEXT-GENERATION of RF integrated circuit designs will always be challenged by an increased number of trouble spots, such as the electromagnetic (EM) coupling among the down-scaled individual devices to be integrated on one chip. EM field coupling is becoming too strong to be neglected. It causes extra design iterations, over-dimensioning or complete failures, unless appropriate solutions are found to resolve these design issues. To address these problems, an European research project entitled: *Comprehensive High-Accuracy Modelling of Electromagnetic Effects in Complete Nanoscale RF blocks—Chameleon-RF* (www.chameleon-rf.org) was started within FP6/IST, as well as a national project: CEEX/nEDA (http://neda.lmn.pub.ro). The general objective of the Chameleon-RF project is that of developing a methodology and prototype software tools that take layout descriptions of the complete RF functional blocks that will operate at frequencies up to 60 GHz and transform them into sufficiently accurate, reliable electrical simulation model (described by appropriate Spice circuits), taking EM coupling effects as well as technology variability into account.

The geometric complexity of nowadays designs can be handled only by domain decomposition (DD). It is the technique of choice in parallel EM computation, and it assumes the existence of a splitting of the computational domain into sub-domains which can be interfaced in many ways. Interface relaxation (IR) is a frequently encountered iterative form of DD solvers [1]. It assumes a splitting of the domain into a set of non-overlapping sub-domains and considers the associated PDE problems defined on each of them. IR can be implemented by traditional numerical methods which can vary from sub-domain to sub-domain. These subproblems are coupled by means of relaxation mechanisms on the interfaces. Several interface relaxation methods, considered from the domain decomposition viewpoint, like the Schwartz method, the Poincaré–Steklov method, the Schur complement, etc., can be found in the literature [2].

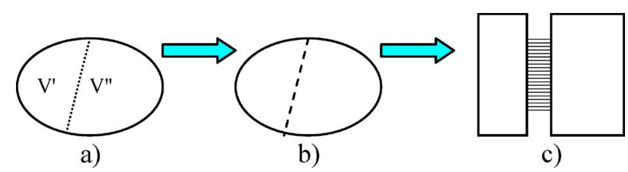

Fig. 1. Domain partition: (a) exact (b) with hooks (c) circuit.

The novelty of this paper is that it analyses the convergence properties of new interface approximations, based on electric and magnetic hooks, which were proposed for the first time in [3] to describe the parasitic EM interaction between components in RF-ICs.

II. Interface Conditions in Static Cases

In order to extract the equivalent lumped RLC parameters of several passive integrated components, the equations of the static EM fields have to be solved. In the case of static electric fields [electro-static (ES) and steady-state electric-conduction (EC)], the electric potential V satisfies an elliptic PDE, which in homogeneous domains becomes the Laplace's equation. The computational domain is conventionally partitioned in simpler and smaller sub-domains [see Fig 1(a)]. The conditions on virtual interfaces between these sub-domains (assuming same materials on both sides) are

$$V'(P) = V''(P), \quad \frac{dV'}{dn} = \frac{dV''}{dn}. \tag{1}$$

Similar relationships hold for the magnetostatic (MS) field, for the magnetic scalar potential.

An important improvement of the numerical efficiency is obtained if the interaction between sub-domains by means of their common interfaces can be described by using a reduced number of degrees of freedom (DoFs). In this context, the electric circuit element (ECE) boundary conditions are a natural choice [4]. In

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this respect, on each interface n equi-potential surfaces called "terminals" are defined, on which $V = v_k$. On their complement $dV/dn = 0$ stands [see Fig. 1(b)]. In these conditions, each sub-domain is compatible with an n-polar external circuit [see Fig. 1(c)]. From (1) the interconnection relations for voltages and currents of terminals are obtained

$$v'_k = v''_k; \quad i'_k = -i''_k; \quad k = 1, 2, \ldots, n. \tag{2}$$

The admittance matrix $\mathbf{Y}$ ($\mathbf{G} + s\mathbf{C}$ in the electro-quasi-static (EQS) field regime), of the equivalent circuit represents a discrete approximation of the Poincaré–Steklov operator, when V is piece-wise-constant. Solution uniqueness in Dirichlet boundary conditions (V known => $\mathbf{E}_t$) implies the correct definition of this operator and therefore of its discrete approximation $\mathbf{Y}$. In EC regime, the power transferred through the interface from a sub-domain to its neighbor is

$$P = \int_D \sigma(\nabla V)^2 dv = \mathbf{i}^T\mathbf{v} = \mathbf{v}^T\mathbf{i} = \mathbf{v}^T\mathbf{Y}\mathbf{v}. \tag{3}$$

Regardless of the numerical method used for PDE discretization (e.g., first-order FEM, BEM, of FIT), a system of linear equations $\mathbf{Ax} = \mathbf{b}$ is generated. $\mathbf{Y}$ is the Schur complement of the block in matrix $\mathbf{A}$ corresponding to the grid nodes which are not on the interface. Terminals act as hooks between sub-domains. They allow independent meshing, and even using independent PDE or numerical method in each sub-domain. If adjoin sub-domains have conformal aligned meshes, the number of hooks can be increased up to the limit when each node on the interface is an independent terminal. In this degenerate case, the interface does not perturb the field solution, thus being numerically transparent. According to the convergence theorem, in correct discretizations, the numerical solution tends to the exact one, when the norm of the mesh goes to zero. In these conditions, *the number of hooks tend to infinity and the interface becomes perfectly transparent*.

In order to illustrate this conclusion, the simple case of a CPW transmission line [5] was numerically solved. The p.u.l. capacitance was extracted from the 2-D-ES field in the entire domain, computed by FIT with 100 × 243 nodes [6]. Afterwards, n hooks were placed on the interface between silicon and SiO2. The relative error between the two numerical results decreases to zero w.r.t. n as shown in Fig. 2. If 2% is an acceptable error, 8 hooks (30 times less the maximal number) are enough.

According to [7], the sensitivity of the equivalent circuit with n terminals (e.g., the entries Gij in the conductance matrix) w.r.t. a geometric parameter p which change the position of the terminal surface S_p is

$$S = \frac{\partial G_{ij}}{\partial p} = \frac{1}{V_0^2}\int_D \frac{\partial \sigma}{\partial p}\mathbf{E}\cdot\underline{\mathbf{E}}d\Omega = \frac{\sigma}{V_0^2}\int_{Sp} \mathbf{E}\cdot\underline{\mathbf{E}}d\Omega \tag{4}$$

where $\underline{\mathbf{E}}$ is the adjoint electric field. Placement of a hook-terminal means metallization of a plate S_p on the sub-domain boundary. The error induced by terminal extension with δp is

$$\delta G_{ij} = S\delta p \cong \frac{J \cdot \underline{E}}{V_0^2}\delta p \tag{5}$$

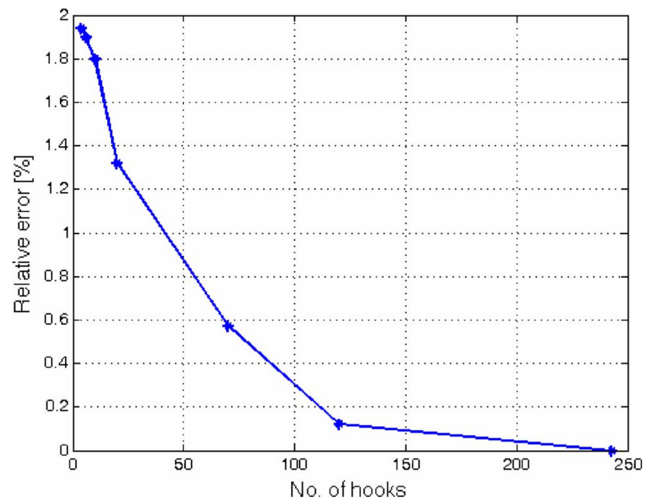

Fig. 2. Relative error of C [%] versus number of hooks.

where $\mathbf{J}$ is current density produced by $V_i = V_0$ and $\underline{\mathbf{E}}$ is the field produced by $V_j = V_0$ The dot product is evaluated in the middle of the link with size δp, which will be metallized.

III. Electric and Magnetic Hooks in FW Field

The interface conditions for full wave (FW) field are

$$\begin{aligned} \mathbf{E}'_t(P) &= \mathbf{E}''_t(P), \quad J'_n + dD'_n/dt = J''_n + dD''_n/dt \\ \mathbf{H}'_t(P) &= \mathbf{H}''_t(P) \quad B'_n(P) = B''_n(P) \end{aligned} \tag{6}$$

for any P on the interface. Although the uniqueness of FW field requires only $\mathbf{E}_t$ or $\mathbf{H}_t$, (6) suggests the necessity of both electric and magnetic hooks, for a proper approximation of interaction through interface. electro-magnetic circuit element (EMCE) provides suitable boundary conditions for a robust approximation of this interaction. By definition, EMCE *is a simply connected domain D bounded by a fixed closed surface Σ composed by n' disjoint parts $S'_1, S'_2, \ldots, S'_{n'}$, called* ***electric terminals (hooks)*** *and n'' disjoint parts $S''_1, S''_2, \ldots, S''_{n''}$, called* ***magnetic terminals (hooks)*** *on which*

$$\mathbf{n}\cdot \mathrm{curl}\mathbf{E}(P,t) = 0, \quad \forall P \in \sum - \cup S''_k \tag{7}$$

$$\mathbf{n}\cdot \mathrm{curl}\mathbf{H}(P,t) = 0, \quad \forall P \in \sum - \cup S'_k \tag{8}$$

$$\mathbf{n}\times \mathbf{E}(P,t) = \mathbf{0}, \quad \forall P \in \cup S'_k \tag{9}$$

$$\mathbf{n}\times \mathbf{H}(P,t) = \mathbf{0}, \quad \forall P \in \cup S''_k \tag{10}$$

where $\mathbf{n}$ is the unitary vector, orthogonal to Σ in P.

Condition (6) ***prevents inductive couplings*** through element boundary, between inside and environment, excepting for the magnetic terminals. This condition can be complied by enlarging the boundary Σ, so that the magnetic field has a negligible normal component or it may be considered perpendicular to the magnetic terminals. Condition (7) ***implies absence of conductive and capacitive couplings*** through the element boundary, excepting for the electric terminals. Condition (8) ***excludes variation of the electric potential over every electric terminal***, allowing its coherent connection to a node of an external electric circuit. It is automatically satisfied, if electric terminals are perfect conductors (e.g., having ideally

$\sigma = \infty$). Consequently, the current lines are orthogonal to the electric terminal surfaces. Finally, condition (9) ***excludes variation of the magnetic potential over every magnetic terminal***, allowing its connection to a node of an external magnetic circuit. It is automatically satisfied, if these terminals are made by perfect magnetic materials (e.g., having ideally $\mu = \infty$). Consequently, the magnetic lines are orthogonal to the magnetic terminal surfaces. With these boundary conditions, the interaction between EMCE and its environment is completely described by $2(n' + n'')$ scalar variables, two for each terminal. They are: the terminal current and voltage for the electric terminals and flux and magnetic voltage for the magnetic terminals.

The EM field is unique in these boundary conditions if terminals are excited in voltage $\mathbf{v}$, $\mathbf{v}_m$. Therefore, the operational hybrid transfer function of each sub-domain

$$\begin{bmatrix} \mathbf{i} \\ S\varphi \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{v}_m \end{bmatrix} \tag{11}$$

is well defined, even in the FW regime. Here, the output signals are currents $\mathbf{i}$ of electric terminals and time derivative of flux $s\varphi$ of magnetic terminals, $\mathbf{Y}_{11}$ and $\mathbf{Y}_{22}$ are electric and magnetic admittance, respectively, while $\mathbf{Y}_{12} = \mathbf{Y}_{21}$ (due to reciprocity) describes the EM interaction inside EMCE, e.g. voltage induced by the time variation of the injected magnetic flux.

For each electric terminal k, its ***current*** is defined as the magnetic field loop-integral

$$i_k(t) = \int_{\Gamma_k} \mathbf{H} \cdot d\mathbf{r} \tag{12}$$

where $\Gamma_k = \partial S'_k$ is the closed curve—boundary of S'_k surface (representing the total current).

For each electric terminal k, the ***voltage*** is defined as the integral:

$$v_k(t) = \int_{\mathbf{C}_k} \mathbf{E} \cdot d\mathbf{r} \tag{13}$$

along an arbitrary curve C_k, included in $\Sigma - \bigcup S''_k - \bigcup T''_k$ which is a link between a point on S'_k and a point on S'_n. Here T''_k is a path belonging to Σ which links a point on S''_k with a point on S''_{k+1}.

For each magnetic terminal k, its ***flux*** is defined so that its time derivative is

$$\varphi_k(t) = \int_{\Gamma_k} \mathbf{E} d\mathbf{r} \tag{14}$$

where $\Gamma_k = \partial S''_k$ is the contour—boundary of S''_k surface.

For each magnetic terminal k, the ***magnetic voltage*** is defined as the integral

$$v_{mk}(t) = \int_{C_k} \mathbf{H} d\mathbf{r} \tag{15}$$

along an arbitrary path C_k, included in $\Sigma - \bigcup S'_k - \bigcup T'_k$ which is a link between a point on S''_k and a point on S''_n. Here T'_k is a path in Σ, which links a point on S'_k with a point on S'_{k+1}.

The following uniqueness theorem is fundamental for the correct formulation of the EM field problem. *The EM field problem associated to the EMCE with boundary conditions* (6)–(9), *zero initial condition and having some terminals excited in known voltage and the rest in known current/flux has a unique solution:* $\mathbf{E}(M,t)$, $\mathbf{D}(M,t)$, $\mathbf{B}(M,t)$, $\mathbf{H}(M,t)$, $\mathbf{J}(M,t)$, $\rho(M,t)$ *of the Maxwell equations for* $\forall M \in D$, $t > 0$ *and therefore EMCE has a unique response*.

This theorem is a direct consequence of the expression of electromagnetic power transferred by means of its boundary from the outside to the inside of any EMCE

$$P = \int_{\Sigma} (\mathbf{E}_t \times \mathbf{H}_t)\mathbf{n} dS = \mathbf{v}^T\mathbf{i} + \mathbf{v}_m^T \frac{d\varphi}{dt}. \tag{16}$$

This is the particular form of the *Timotin*'s theorem, applied to a simple connected EMCE [4]. Electric hooks describe the conductive and capacitive interaction between sub-domains, while magnetic/inductive interaction is described by magnetic hooks. One should be aware that according to the EMCE definition, the electric and magnetic hooks can not be overlapped. External electric circuits are connected to electric terminals, while inductive coupling is described by magnetic circuits, connected to magnetic terminals, as in the VPEC technique [8].

The convergence property of static hooks is still valid for FW, at least for numerical methods based on Yee grids, such as FDTD or FIT. Any interface with maximal number of hooks (e.g., each node of the primary grid is an electric hook and each node of dual grid is magnetic hook) is numerically transparent for the EM field. Details about numerical implementation of these interface conditions are given in [9].

The numerical test using FW field consisted of a structure with two coplanar U-shaped coupled conductors which was numerical modeled by FIT. On the interface (the symmetry plane) between sub-domains a maximal number of hooks were considered in the following cases: only electric hooks; only magnetic hooks; and both electric and magnetic hooks. The frequency characteristics obtained for these three cases are given in Fig. 3. At low frequency, the magnetic hooks are more important while at high frequency, capacitive effects became significant. The interface with all electric and magnetic connectors is numerically transparent, simulation of full domain producing the same result as interconnected partitions [9]

The parallelism can be computationally exploited in two manners. On the one hand, the model of each sub-domain can be extracted by an independent CPU or cluster of CPUs. On the other hand, the frequency characteristics can be extracted in parallel, each frequency being distributed to a different CPU. Using the computed frequency characteristic, the SPICE equivalent model of each sub-domain are extracted by VectorFit [10]. All these numerical experiments were carried out on a cluster of 12 standard PCs connected in a LAN by FastEthernet.

IV. Conclusion

The domain partitioning and hook technique was successfully applied in noise propagation studies, as a technique to approximate the interface conditions in the modeling of parasitic coupling between IC components and their EM environment. In this

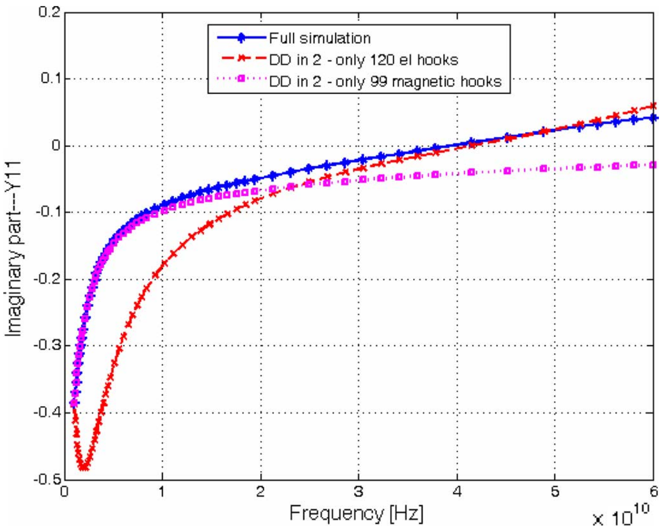

Fig. 3. Frequency characteristics with several kinds of hooks.

case, the precise distribution of EM field is not relevant, only approximate value of global quantities matters. Each sub-domain was numerically analyzed considering its own field regime: FW in SiO_2 layer, MQS in conductors, EQS+MS in the Si substrate, and ES+MS in the air above chip.

The hooks technique has practical importance when their number is reduced to 1,. . .,10. With such values, the sub-domains having different shapes can be modeled independently and in parallel. Afterwards the reduced size models (represented as matrices—frequency dependent circuit functions, state equations or reduced order Spice circuits) are interconnected, aiming to obtain a model for the global system. Unlike DD, which is basically an iterative process, in the proposed approach of Domain Partitioning (DP), due to the reduced numbers of hooks, the interface iterations can be removed and the resulted technique is a "direct," not an iterative one, as DD is. The global modeling effort is then reduced, replaced by the independent model extraction for each sub-domain.

Iterations have to be removed due to IC designers' requirement, for the library of components models being *a priori* generated, without knowing their final position on chip. Therefore, the classical DD approach cannot be applied in the design of the complex RF integrated circuits. For this reasons, the challenge to reduce the number of hooks becomes imperative. Otherwise, the EM field in nowadays RF-ICs working up to 100 GHz will become insolvable, due to its geometric complexity.

In order to identify the hooks, nodes on interface have to be merged in a minimal number of clusters, so that approximation error (5) be kept below an acceptable level. Hence, the pseudo-optimal hooks identification, problem related to so called "terminal reduction" [11] is formulated as a discrete optimization problem.

In addition to the clustering algorithms [11], heuristic rules may be also applied for this reduction. For instance, it is recommended the placement of magnetic hooks in the holes of spiral inductors and of electric hooks near to conductors, in order to allow a proper modeling of capacitive couplings. As a general rule, the interface should be as close as possible to a constant potential surface, orthogonal to the field lines. For instance, symmetrical or very elongated domains can have only one hook on the middle cross-section. In FW, the process of hooks identification is very similar to the strategic game of Go, because the target is to maximize the size of non-overlapping clusters of electric (white) and magnetic (black) grid nodes. Being a NP problem, heuristic solutions cannot be competed.

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