Simplification by Pruning as a Model Order Reduction Approach for RF-MEMS switches

Gabriela Ciuprina, Daniel Ioan, Aurel Sorin Lup, Luis Miguel Silveira, Anton Duca and Michael Kraft*†‡§

Abstract

Purpose This paper advocates the extraction of reduced order models by using a physics aware simplification technique.

Design/methodology/approach The reduced model is built progressively, by increasing the complexity of the physical model. The approach starts with static analyses and continues with dynamic ones. Physical phenomena are introduced sequentially in the reduced model whose order is increased until accuracy, computed by assessing forces that are kept in the reduced model, is acceptable.

Findings The technique is exemplified for RF-MEMS switches, but it can be extended for any device where physical phenomena can be included one by one, in a hierarchy of models. The extraction technique is based on analogies that are carried out both for the multiphysics and the full-wave electromagnetic phenomena, as well as for their couplings. In the final model the multiphysics electromechanical phenomena is reduced to a system with lumped components with nonlinear elastic and damping forces coupled with a reduced system with distributed and lumped components which keeps the electromagnetics.

Originality/value Contrary to order reduction by projection, this approach has the advantage that the simplified model can be easily understood, the equations and variables having significance for the user. The novelty of the proposed method is that, being tailored to a specific application, it is able to keep physical interpretation inside the reduced model. This is the reason why, the obtained model has an extremely low order, much lower than the one achievable with general state-of-the-art procedures.

Keywords MEMS modeling, Multiphysics, Computational electromagnetics, Model order reduction

Paper type Research paper

1 Introduction

The design and simulation of complicated systems involving many physical phenomena use macromodels of their constitutive parts. Switches are used in many micro-electro-mechanical systems (MEMS), and their macromodels need the computation of equivalent parameters reflecting both the behavior of the device when it switches and its efficiency of transmission/blocking the RF signal [18]. The easier the simulation of the device macromodel, the faster the simulation of the system level realization. This paper illustrates that if a reduction procedure is tailored for a specific device by taking into consideration physical phenomena, the obtained macromodel is smaller and needs less computational effort than one obtained from general reduction algorithms.

In their transition from one stable state to the other, most radio-frequency (RF) MEMS switches are actuated by an electrostatic force. This force depends nonlinearly on the armature displacement. Thus, even if the partial differential equations (PDEs) that describe the electrostatic and structural fields

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are linear, their coupling is nonlinear and consequently, the whole dynamic system that describes the relationship between its input (electric actuation voltage) and its output (displacement of the membrane) is nonlinear. After discretization, for instance with the finite element method (FEM), the set of coupled equations generates a system of ODEs of high order, which has to be reduced in order to be useful for the design. The difficulty of reducing the order of these discrete models comes from the non-linearity of the system [13, 6].

Some approaches combine the use of analytic formulas obtained in simple cases (such as a parallel plate capacitor) for which corrections are applied, followed by heuristic fitting based on measurements or simulations [12, 8]. According to the terminology from [5], these are *physics-based models*, their main advantage being their use for scaling studies and optimization.

Another recent study proposes a closed-form expression for the pull-in voltage based on the use of the differential evolution optimization algorithm together with the FEM simulation data for hundreds of configurations [1]. Other approaches are mathematically rigorous, e.g. there is an impressive literature of model order reduction (MOR) methods for linear systems [17], either based on projection of the original state space on a lower dimensional space, such as Krylov subspace methods, truncated balanced realizations or proper orthogonal decomposition (POD) or based on fitting the response, such as vector fitting (VF). POD and VF are data driven, they lose the structural information of the reduced system, being black-box approaches.

Some MOR methods for linear systems were extended to the nonlinear case, e.g. by linearizing the system before applying Krylov projection [2, 14]. The weakness of a such approach is that the reduced model is accurate only around the initial operation point of the nonlinear device. POD is the only one among the methods mentioned above which is intrinsically appropriate for the reduction of nonlinear systems, but it is not as effective as its linear version [17]. Consequently, approaches that were especially tailored for nonlinear systems have been proposed [7]. A successful approach, called Trajectory Piece-Wise Linear (TPWL) [13] reduces the system by piece-wise linearisation about several points of the state space trajectory. Another successful method is Discrete Empirical Interpolation (DEIM), which is an extension of POD carefully devised to make the computations efficient [3]. In [11] the MOR methods for MEMS are seen either node-oriented or domain-oriented. Node-oriented methods search for reduced order approximations of the matrices that describe the behavior of nodes, whereas the domain-oriented methods are based on modal analysis. The method we propose combines these two approaches. In [7] the terminology "Simplification by pruning" (SP) is used to describe model reduction based on physical principles, less based on mathematical rigorousness as projection methods are. This simplification is based on understanding the dynamics of the model so that the analysis is easy and the simulation is fast. For this, it is important to identify the variables that have a minor impact on the system behavior and eliminate them. Finally, there are very few variables left, with a direct physical interpretation. The reduction process incorporates methods that refer to different parts or properties of the studied model. Some parts may require even a static analysis. After each simplification, the accuracy of the model has to be checked. Contrary to order reduction by projection, this approach has the advantage that the simplified model can be easily understood, the equations and variables have significance for the user. Even if the name *simplification by pruning* may suggest the contrary, in this simplification, one starts with a model of minimal order, which is increased until the approximation error is acceptable.

2 SP-MOR for multiphysics phenomena of switches

In [4] we proposed a method to extract a macromodel for RF-MEMS switches that included the static coupled structural-electric behavior of the switch and its RF behavior. In [9] we generalized this static reduction to a multi-body system consisting of p point masses. This static reduction allows only the extraction of the pull-in voltage. Here, we extend the procedure to include dynamic phenomena. Moreover, the algorithm is generalized so that to reduce the device to a set of p point masses m_j, a set of springs and a set of dampers.

Considering the dimensions of the device, the electrostatic (ES) field between the armatures can be assumed normal to the fixed electrode. The electric field reduction is based on the assumption that the field is piece-wise uniform and thus the ES forces are concentrated in p points, and they are normal to the fixed electrodes. The approach is illustrated in Fig.1 where only dampers placed between the point masses and the fixed armature were considered. Such a simplification is based on physical considerations, only the damping forces that are normal to the fixed armature being relevant.

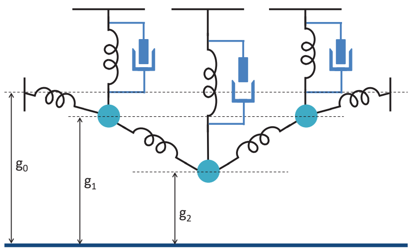

Figure 1: Higher order physics inspired reduced order model (here $p = 3$). Mechanical view.

If we denote by $\mathbf{M}$ the diagonal matrix holding the lumped masses, $\mathbf{B}$ the matrix holding the damping coefficients, $\mathbf{K}_1$ and $\mathbf{K}_3$ the matrices holding the elastic coefficients, the equation of motion of the p points of interest $\mathbf{z}$ is:

$$\mathbf{M}\frac{\mathrm{d}^2\mathbf{z}}{\mathrm{d}t^2} + \mathbf{B}\frac{\mathrm{d}\mathbf{z}}{\mathrm{d}t} + \mathbf{K}_1\mathbf{z} + \mathbf{K}_3\mathbf{z}^\mathbf{3} = \mathbf{f}_{\mathrm{ES}}(u, \mathbf{z}), \tag{1}$$

where $\mathbf{f}_{\mathrm{ES}}$ is the vector of electrostatic forces lumped in p points of the armature and $\mathbf{z}^\mathbf{3}$ is a notation for a vector with the entries z_i^3, $i = 1, \ldots, p$. The forces in $\mathbf{f}_{\mathrm{ES}}$ depend on the applied voltage $u(t)$ and the displacement of the neighborhood of the application point. Linear elastic forces correspond to the equation with $\mathbf{K}_1 = \mathbf{K}$, $\mathbf{K}_3 = \mathbf{0}$. In the reduced model various structures for the elastic matrices were considered: (1) tridiagonal and symmetric, (2) tridiagonal and nonsymmetric, (3) full and nonsymmetric. Each structure of the elastic matrix corresponds to a topology of the similar electric circuit, equivalent to a certain elimination of nodes from the discrete equivalent circuit. The extraction algorithm is successful if the final value of p is much less than the number of degrees of freedom used in the numerical field simulation from which the reduced model is extracted.

The procedure has four steps procedure. The first two steps aim to approximate the behavior of the nonlinear system in the static regime, i.e. find a minimum number of parameters that describe the way in which the equilibrium position of the armature, monitored in a low number of points, depends on the applied voltage. The third step aims to approximate the nonlinear system in a dynamic regime without damping, finding a minimum number of parameters that describe accurately enough the resonant frequencies. The final step aims to approximate the dynamic behavior in a regime with damping. Thus, from the physics phenomena point of view the reduced model is augmented sequentially, each step relying on the previous one. From the reduced order model point of view, the complexity is also increased progressively, by doing only postprocessing computations using the initial data obtained from three type of simulations of the full order model: static, dynamic without damping and, finally, with damping. The details of this physics aware model reduction are given below.

Step 1: ***Compute lumped electric forces from static simulation.*** A strongly coupled static structural-electrostatic simulation is carried out, from which the electrostatic forces in p chosen points are extracted. Considering the real dimensions of the device, the electric field between the armatures is assumed normal to the fixed armature, and piece-wise uniform. Thus, the inverse of the lumped capacitance C_j that corresponds to a point j depends linearly on the displacement z_j of that point $C_j = 1/(c_{1,j}\, z_j + c_{2,j})$. By using the generalized forces theorem, the ES force that acts on the point j is:

$$f_{\mathrm{ES}}(u, z_j) = -c_{1,j}\, z_j/(c_{1,j}\, z_j + c_{2,j})^2 u^2. \tag{2}$$

During this step, a set of coupled static simulations are carried out, for various values of u, up to the pull-in. Thus, several sets of pairs z_j, C_j are obtained and the coefficients $c_{1,j}$ and $c_{2,j}$ are obtained by fitting each $1/C_j$ to a linear dependence $c_{1,j} z_j + c_{2,j}$.

Step 2: ***Extract effective elastic coefficient matrices.*** Since in the static simulation the electric forces are equal to the elastic forces, the values obtained at step 1 are used by a first or third order least square fitting of forces versus displacements. Thus, either $\mathbf{K}$ or the pair $\mathbf{K}_1, \mathbf{K}_3$ are obtained. The matrix $\mathbf{K}$ can be extracted by moment matching techniques, but here we preferred the fitting approach which can be easily extended to the nonlinear-cubic case. Numerical results obtained for $p = 1$, thus 2 degrees of freedom (DOFs), showed that the cubic dependence is able to predict accurately both the pull in voltage (error less than 1%), and the dependence of the displacement with respect to the applied voltage [4]. However, when features specific to the dynamic behavior are of interest, the order should be increased at least to $p = 3$ (4 DOFs). In this case the structure of the elastic matrices has also to be chosen among the possibilities described above.

Step 3: ***Extract lumped mass matrix.*** The effective mass matrix $\mathbf{M}$ is extracted from a time domain simulation without damping, for an input excitation that is less than the pull-in voltage, such that oscillations are visible in the membrane movement. From this undamped response, the displacements $\mathbf{z}$ of the points of interests are recorded in time and the inertial force vector is extracted:

$$\mathbf{f}_{\mathrm{in}}(u, \mathbf{z}) = \mathbf{f}_{\mathrm{ES}}(u, \mathbf{z}) - \mathbf{K}_1\mathbf{z} - \mathbf{K}_3\mathbf{z}^{\mathbf{3}}. \tag{3}$$

Each component of the extracted inertial force $\mathbf{f}_{\mathrm{in,j}}$ is fitted with a first order polynomial in the local acceleration $\mathrm{d}^2 z_j/\mathrm{d}t^2$. The accuracy of the fit is given by the square of the sample correlation coefficient and it increases with the model order. This quantity is a criterion to chose an appropriate order. A supplemental condition to validate the accuracy of the reduced order model extracted so far can be related to the behaviour to other type of excitations, e.g. to a test signal that would close the switch. This method of finding the lumped masses is equivalent to modal analysis.

Step 4: ***Extract lumped damping matrix.*** The effective damping coefficient matrix $\mathbf{B}$ is extracted from a time domain simulation with damping. From this damped response, the displacements $\mathbf{z}$ in the points of interests are recorded in time and the damping force vector is extracted:

$$\mathbf{f}_{\mathrm{d}}(u, \mathbf{z}) = \mathbf{f}_{\mathrm{ES}}(u, \mathbf{z}) - \mathbf{K}_1\mathbf{z} - \mathbf{K}_3\mathbf{z}^{\mathbf{3}} - \mathbf{M}\mathrm{d}^2\mathbf{z}/\mathrm{d}t^2. \tag{4}$$

The vector of damping forces is fitted with respect to the vector of local velocities. If each component of the extracted damping force $\mathbf{f}_{\mathrm{d,j}}$ is fitted with respect to the local velocity $\mathrm{d}z_j/\mathrm{d}t$, then the matrix $\mathbf{B}$ is diagonal and corresponds to damping forces that are normal to the fixed electrodes. Numerical tests show that the inclusion of nondiagonal terms in $\mathbf{B}$ does not improve the accuracy of the reduced model. However, in order to obtain sufficiently accurate results, the dependences between the lumped damping forces and the local velocities have to be nonlinear. In the literature, for instance in [15], the damping matrix is computed by using the Rayleigh approximation, i.e. it is a weighted linear combination of the matrices $\mathbf{M}$ and $\mathbf{K}$, thus defined by two real coefficients which are the weights. Physical considerations show that the damping due to the air squeezing is more complicated.

Steps 3 and 4 need a certain excitation signal $u(t)$ (fitting signal). Step 3 needs an excitation signal that makes the switch oscillate, whereas step 4 needs an excitation signal that makes it close. After step 4, the complete dynamical model is obtained, able to describe both static characteristics (pull-in voltage) and dynamical ones (closing time).

There are several quantities that measure the success of each step such as: fitting of the capacitance (step 1), relative error between the electric force of the reduced model and the electric force of the full model (step 1), fitting of the elastic force (step 2), relative error of the pull-in voltage of the reduced model with respect to the full model (steps 1 and 2), approximation of the mass (step 3), approximation of the damping coefficient (step 3), relative error of the closing time by comparing the results of the reduced system (obtained for a step voltage greater than the pull-in voltage - new testing signal), with the real response of the system. The algorithm goes to step 4 only after steps 1 to 3 are accurate enough in terms of three criteria: accuracy of the electric force concentration, pull-in voltage, mass fitting. The accuracy is computed as a relative difference between the result given by the reduced order model and the corresponding quantity of the full order model. The mentioned criteria are useful in establishing the model accuracy. If this is not satisfactory for a certain complexity order, then this has to be modified and the model refined, until the requirement imposed to the accuracy is obtained. In fact, the most time consuming computations are in the simulations of the full order model, but even here efficient techniques can be used, such as those obtained by combining finite elements for the structural domain with boundary elements for the electrical domain, as in [16]. When carrying out steps 1 and 2 which will decide the order of the reduced model, several low order models, with various characteristics (e.g. different structures of the elastic matrix) can be easily generated and tested, so that the best one is finally selected. Details on this aspect are given in section 3.

3 Results

In order to see if this physics inspired algorithm is able to identify the parameters of a reduced system of extremely low order, corresponding to $p = 1$, so with only 2 DOFs (displacement and velocity of the point expected to realize the contact), a first test configuration, of cantilever type. with an initial air gap $g_0 = 2\,\mu$m, length $300\,\mu$m, width $20\,\mu$m, thickness $t = 2\,\mu$m, Young modulus $E = 153$ GPa, Poisson's ratio $\nu = 0.23$, mass density 2330 kg/m^3.

The computational domain includes half of the cantilever and the simulations of the full model (with 433051 DOFs) was carried out by using the FEM of COMSOL[1]. For steps 3 and 4 of the algorithm a step voltage to 5V was applied. The first condition to check is whether the extracted reduced order model (ROM) is able to approximate the full order model (FOM) excited with the same signal.

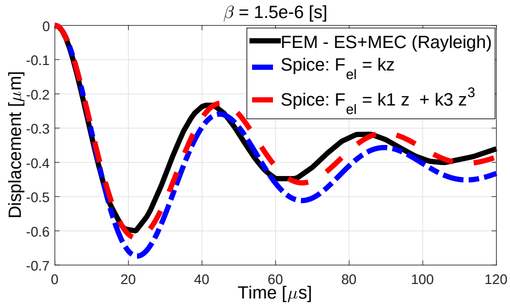

Figure 2: Dynamic simulation for a step voltage of 5V applied to a model with an imposed local Rayleigh damping of $\beta_s = 1.5 \cdot 10^{-6}$s.

Besides the electrostatic field, a Rayleigh damping β_s was included explicitly in the FEM elastic material model. Such damping is usually expressed as $b_s = \alpha_s m + \beta_s k$, where α_s is the mass-damping coefficient and β_s is the stiffness-damping coefficient. We took $\alpha_s = 0$, imposed various values for β_s and extracted the global β to be included in the macromodel ($\mathbf{B} = \beta\mathbf{K}$). The reduction was thus from order 433051 to order 2 and the simulation of the equivalent circuit was carried out in SPICE. Fig. 2 shows the comparison of dynamic responses and Table 1 holds quantitative results. Even if the relative difference between the imposed β and the extracted one is relatively high (20% in some cases), the dynamical responses are satisfactory, again better when a cubic dependence of the elastic force with respect to the displacement is considered (relative error less than 2%).

This small error validates the fitting carried out by the algorithm. We note again the importance of the nonlinear, cubic term, which increases the accuracy of the model. It represents the effect of the spatial distribution of the displacement, considering that in the ROM the displacement is described by the gap as sole geometric parameter.

Table 1: Extraction of Rayleigh Damping Parameter

Imposed β_s [s]	Extracted β [s]	Rel diff. [%]	Computed $b = \beta * k$ [g/s]	Rel.er.[%] FEM-SPICE use k	k_1, k_3
6.0e-5	7.1e-5	19.2	1.70e-2	1.2	0.082
6.0e-6	7.3e-6	21.5	1.73e-3	2.1	0.074
1.5e-6	1.8e-6	18.7	4.23e-4	12	0.55
6.0e-7	6.8e-7	13.0	1.61e-4	26	1.4

The reduction of a second test case having a bridge configuration (Fig. 3) to $p = 1$ was successful only to find the stiffness coefficients, but failed in the extraction of the effective mass. This is due to

[1]This benchmark is taken from COMSOL library examples.

the fact that the inertial force cannot be accurately fitted with a first order dependence with respect to the acceleration. In this case the remedy consists of increasing the number of mass points of the reduced device.

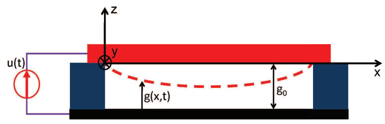

Figure 3: Bridge benchmark.

To better investigate this aspect, the bridge benchmark suggested in [13] was used. It is a polysilicon beam, of length $l = 610\,\mu$m, width $w = 40\,\mu$m and height $h = 2.2\,\mu$m suspended over a silicon substrate, the initial gap being $g_0 = 2.3\,\mu$m. Since $l \gg w$ the deflection is assumed uniform across the width. A full model is obtained by considering strongly coupled 1D Euler's beam equation (5) and 2D Reynold's squeeze film damping (6):

$$EI\frac{\partial^4 g}{\partial x^4} - S\frac{\partial^2 g}{\partial x^2} = f_{\mathrm{ES}} + f_d - \rho_l \frac{\partial^2 g}{\partial t^2}, \tag{5}$$

$$\mathrm{div}\left(\left(1 + 6\frac{\lambda}{g}\right) g^3 p(\mathrm{grad}(p))\right) = 12\mu \frac{\partial (pg)}{\partial t}, \tag{6}$$

where $g(x,t)$ is the unknown gap, $E = 149\,\mathrm{GPa}$ is the Young modulus, $I = wh^3/12$ is the inertial moment, $S/(hw) = -3.7\,\mathrm{MPa}$ is the initial stress, ρ_l is the per unit length mass ($\rho_l/(hw) = 2330\,\mathrm{kg/m^3}$), $f_{\mathrm{ES}} = -\varepsilon_0 w u^2/(2g^2)$ [N/m] is the per unit length electric force, $f_d = \int_0^w (p - p_a)\mathrm{d}y$ [N/m] is the per unit length damping force, $p(x,y,t)$ is the unknown pressure, $p_a = 1.013 \cdot 10^5\,\mathrm{Pa}$ is the environment pressure, $\lambda = 0.064\,\mu$m is the mean free path of air and $\mu = 1.82 \cdot 10^{-5}\,\mathrm{kg/(m \cdot s)}$. The boundary conditions are: $g(0,t) = g_0$, $g(l,t) = g_0$ for the gap and $p(x,0,t) = p_a$, $p(x,w,t) = p_a$, $\partial p/\partial x(0,y,t) = 0$, $\partial p/\partial x(l,y,t) = 0$ for the pressure. The initial conditions are $g(x,0) = g_0$, $p(x,y,0) = p_a$. A finite difference scheme was used for the spatial discretization and a stiff multi-step ODE solver for time stepping. The initial full order model has 902 DOFs, but it can be made arbitrarily high by making a finer discretization.

Figs. 4 and 5 show results from the static simulation. Table 2 lists the relative errors in the pull-in voltage and the electric forces. For the pull-in voltage, the reduction to one spring is very accurate (relative error of 0.21 %) if a cubic fit for the elastic force is used, but in order to obtain an acceptable accuracy of the electric force in the reduced model, the order has to be increased. Going from $p = 1$ to $p = 3$ reduces the relative error in the force more than five times. All the above results were obtained

Table 2: Relative errors [%] in the static simulation for the pull in voltage and the electric forces.

	$p = 1$	$p = 3$	$p = 5$	$p = 7$
for Vpi (LS1)	7.45	0.735	0.315	0.210
for forces (LS1)	43.9	8.42	6.00	5.46
for Vpi (LS3)	0.210	0.105	0.100	0

with three diagonal, symmetric elastic matrices. If nonsymmetric or even full elastic matrices are used, a similar accuracy can be gained for a lower order (Fig. 6). These results validate the first two steps of the proposed simplification by pruning MOR algorithm and they emphasize the effect of possible choices that can be made. The best results are obtained for $p = 3$, linear elastic force (LS1), nonsymmetrical, three diagonal or full elastic matrix. The results from Figs. 7 and 8 are obtained during the third step. By increasing the order of the model, the accuracy of the extracted mass increases and it can be measured by the degree of correlation between the acceleration extracted from the full order model. For $p = 1$ the correlation is $r^2 = 0.8677$ and for $p = 3$ the correlation is $r^2 = 0.9519$ (Fig. 7).

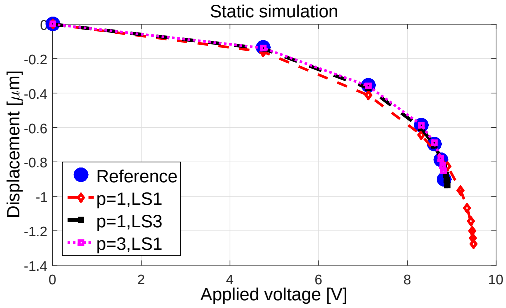

Figure 4: Steps 1&2: Displacement vs. voltage in coupled static simulations. Reduction to $p = 1$ with cubic elastic force gives a relative error in the pull-in voltage less than 1%. So does a reduction to $p = 3$ and linear elastic force. The cubic term in the elastic force is essential for an accurate dramatic reduction to $p = 1$.

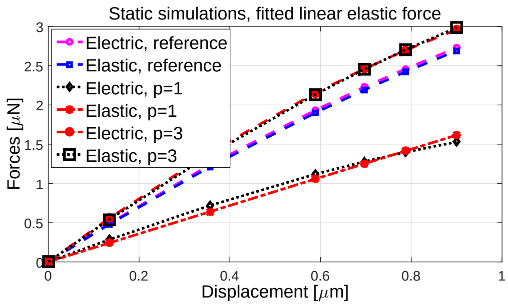

Figure 5: Steps 1&2: The advantage of SbP is that it has physical interpretation inside. Electric and elastic forces can be computed in the reduced model. Increasing p from 1 to 3, reduces the relative error in the forces more than 5 times. A similar error was obtained with a cubic term in the elastic force, and that is why if $p > 1$ only a linear elastic force was considered.

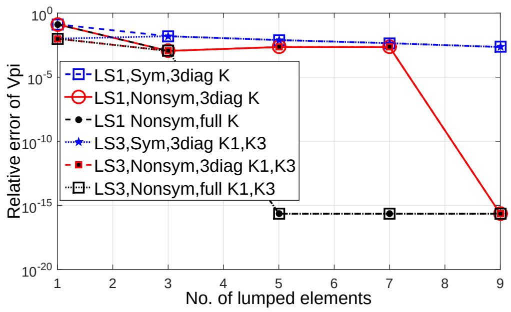

Figure 6: Relative error of the pull-in voltage for various structures of the elastic matrices. The cubic term of the elastic force is useful solely for $p = 1$. For $p > 1$ it is better to use linear elastic force. Various structures of the elastic matrix can be tested with a small computational effort. A value of $p = 3$ gives enough accurate results for the pull-in voltage, the relative errors being less than 1% for all the structures used.

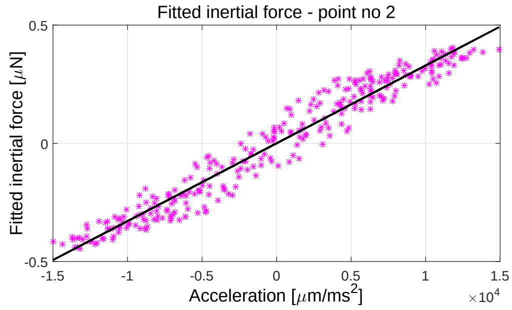

Figure 7: Step 3: Fitting of the inertial force in the case $p = 3$. Correlation: $r^2 = 0.9519$. This is one of the criteria used to establish the value of p.

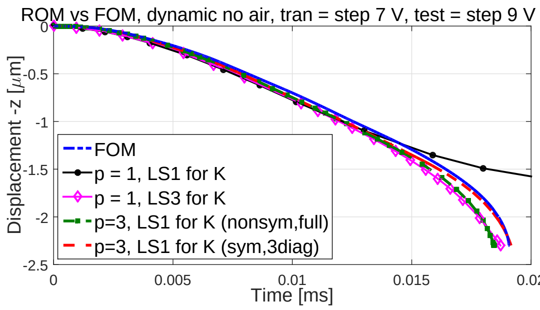

Figure 8: Step 3: Full model vs. reduced model for a test signal that closes the switch. This is the final and most important criteria to chose the value of p.

Fig.8 shows the comparison between the FOM and the ROM for a test voltage that closes the switch. This is the validation for a test signal that was not used for the model extraction. A good agreement (relative error of closing time less than 3 %) is obtained even for $p = 3$ for all the possible structures of the elastic matrices (Table 3). These results validate the third step of the SP-MOR algorithm and emphasize the effect of various choices. A model with $p = 3$ allows us to obtain a closing time with a relative error less than 4 %.

Finally, in the last step the damping forces are extracted. If an input signal that does not close the switch is used for fitting, then a quadratic dependence of the damping force with respect to the velocity is better than a linear fit as can be seen qualitatively in Fig. 9. However, when fitting with such a signal, the answer of the reduced model for a testing signal that closes the switch is too far away from the real behavior as it can be seen in Fig. 11 where the closing time of such a ROM is about or less than half of the closing time of the FOM. This is due to the fact that at higher displacement (greater than about one third of the gap value) the dependence between the extracted damping forces and the velocities have a different pattern, as it can be seen in Fig. 10. Now, a fitting signal that closed the switch was used and a piecewise nonlinear expression that fits the damping-force with respect to the velocity proved to be the best choice. The piecewise non-linear expression consists of two branches, one corresponds to gaps less than one third of the initial gap (range that is covered by the static simulations) and the other corresponds to larger gaps (range beyond the pull-in position). The relative error of the closing time is about 5 % for the reduced model that correspond to $p = 3$ (4 DOFs is symmetry is considered). The effort to make this dependence distributed (i.e. $\mathbf{B}$ nondiagonal) brings no improvement, which proves that the components of the damping forces that are not normal to the bottom electrode have a negligible influence on the dynamics.

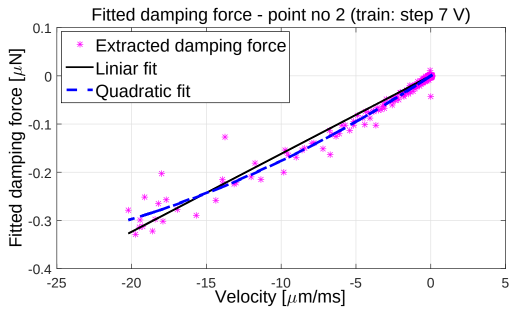

Figure 9: Step 4: Extraction of the damping coefficient by fitting the damping force from a simulation in which the switch does not close.

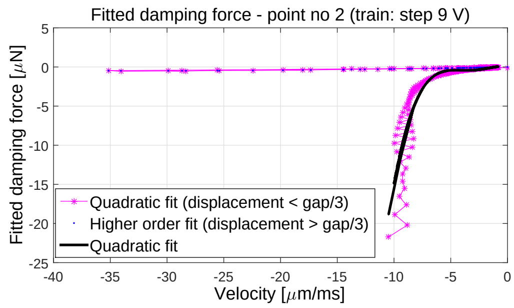

Figure 10: Step 4: Extraction of the damping coefficient by fitting the damping force from a simulation in which the switch closes.

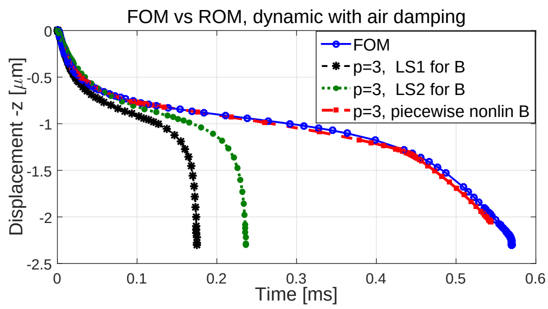

Figure 11: Step 4: Full model vs. reduced model for a test signal that closes the switch. In order to have an enough accurate model, the extraction of the damping coefficient has to be done with a fitting signal that catches the physics involved, i.e. in this case a signal that closes the switch and is able to "feel" the damping up to the contact.

Table 3: FOM vs ROM - dynamic without air simulations, various types of structures for the elastic matrix.

Case	Step 7V f [kHz]	Step 9V t_c [ms]	Step 7V Rel.err.	Step 9V Rel.er.
FOM	27.12	1.90e-2	0	0
$p = 1$,LS1	28.99	N/A	6.8e-2	Not close
$p = 1$,LS3	28.68	1.82e-2	5.7e-2	4.2e-2
$p = 3$,LS1,Sym,3diag.	27.97	1.92e-2	3.1e-2	1.1e-2
$p = 3$,LS1,NonSym,3diag.	28.17	1.86e-2	3.8e-2	2.5e-2
$p = 3$,LS1,NonSym,full	28.17	1.86e-2	3.8e-2	2.5e-2

4 Conclusions

The mechanical phenomena and the RF phenomena of MEMS switches have completely different time scales so that they can be analyzed independently. In this paper we discussed the multiphysics part, while the RF part and the macromodel realization are presented in [10]. Physics inspired reduced order models for the multiphysics behavior are obtained by pruning, starting from field simulations and analogies with systems with lumped components.

The multiphysics model, including air damping effects is reduced to a model characterized by a simpler equation, an ODE with nonlinear right hand side, with coefficients fitted by using the solutions of the PDE. The mobile armature is partitioned in at most 3 segments, each having its position described by a scalar that depends on time. The reduced obtained system consists of p nonlinear differential equations of order 2. It is compact, being described by four lumped matrices, $\mathbf{M}$, $\mathbf{B}$, $\mathbf{K}_1$, $\mathbf{K}_3$, of size $p \times p$ the first two being diagonal, and two vectors of size p which identify the way in which the capacitances depends on the armature position. Besides the nonlinearity generated by the electrostatic force, our model allows a nonlinearity of the elastic force, which is of polynomial type (cubic, without square term) and a nonlinearity of the damping force, which is piece-wise polynomial. The presence of these terms offers solutions that are surprisingly accurate, with a minimum computational effort, especially in the case of a cantilever.

By using such an extraction procedure, a reduced model of very low order is obtained. It cannot be used to predict displacements of the beam under arbitrary excitations but it can give good estimation of important parameters such as the pull-in voltage and the closing time. The SP-MOR method we propose is not a general one, but customized to MEMS devices with elastic membranes, taking into consideration the specific nonlinearities.

It can be placed in-between the methods that process the discretized equations and the methods that process the data. Being tailored for a particular class of problems, it ensures a more efficient reduction and a smaller error than the general methods. Moreover, it saves the physical interpretation after reduction, and it has a controllable accuracy. Since it is tailored for a specific application, the method we propose is not available inside the latest field simulation tools. But, nowadays such software are opening to the users, and therefore writing a code and linking it to the field solver is a common task.

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References

[1] C. Ak and A. Yildiz. A novel closed-form expression obtained by using differential evolution algorithm to calculate pull-in voltage of mems cantilever. *Journal of Microelectromechanical Systems*,

27(3):392–397, 2018.

[2] Zhaojun Bai and Yangfeng Su. Dimension reduction of large-scale second-order dynamical systems via a second-order Arnoldi method. *SIAM Journal on Scientific Computing*, 26(5):1692–1709, 2005.

[3] S. Chaturantabut and D. Sorensen. Nonlinear model reduction via discrete empirical interpolation. *SIAM J. of Scientific Computing*, 32(5), 2010.

[4] G. Ciuprina, A.S. Lup, B. Diţă, D. Ioan, D. Isvoranu, S. Sorohan, and S. Kula. Mixed-domain macro-models for RF MEMS capacitive switches. In *Scientific Computing in Electrical Engineering 2014, Mathematics in Industry*, pages 31–39. Springer, 2016.

[5] G.K. Fedder and Mukherjee. Introduction: Issues in microsystems modeling. In *System-level Modeling of MEMS (T.Bechtold, G.Schrag, L.Feng (Eds))*, pages 1–18. Wiley-VCH Verlag, 2013.

[6] Jeong Sam Han, Evgenii B Rudnyi, and Jan G Korvink. Efficient optimization of transient dynamic problems in MEMS devices using model order reduction. *J. of Micromechanics and Microeng.*, 15(4), 2005.

[7] Mikko Lehtimaki. Dimensionality reduction for mathematical models in neuroscience. Master's thesis, Tampere University of Technology, Tampere, Finland, 2016.

[8] M. A. Llamas et al. Capacitive and resistive RF MEMS switches 2.5D and 3D electromagnetic and circuit modeling. *Electron Devices*, pages 451–454, 2009.

[9] A. S. Lup, G. Ciuprina, D. Ioan, A. Duca, A. Stefanescu, D. Vasilache, and M. Kraft. Parametric reduced order models in static multiphysics analysis of MEMS switches. In *2017 10th International Symposium on Advanced Topics in Electrical Engineering (ATEE)*, pages 513–518, 2017.

[10] A.S. Lup, G. Ciuprina, D. Ioan, A. Nicoloiu, and D. Vasilache. Physics aware macromodels for MEMS switches. under review for COMPEL special SCEE issue.

[11] Ali H. Nayfeh, Mohammad I. Younis, and Eihab M. Abdel-Rahman. Reduced-order models for mems applications. *Nonlinear dynamics*, 41(1):211–236, 2005.

[12] G. M. Rebeiz and G. B. Muldavin. RF MEMS switches and switch circuits. *IEEE Microwave Magazine*, pages 59–71, 2001.

[13] M. Rewienski and J. White. A trajectory piecewise-linear approach to model order reduction and fast simulation of nonlinear circuits and micromachined devices. *IEEE Trans. CAD of Integrated Circuits and Systems*, 22(2):155–170, 2003.

[14] E. B. Rudnyi. MOR for ANSYS. In G. Schrag T. Bechtold and L. Feng, editors, *System-Level Modeling of MEMS (ch.18)*. Wiley-VCH Verlag, Weinheim, Germany, 2013.

[15] E. B. Rudnyi and J. G. Korvink. Model order reduction of mems for efficient computer aided design and system simulation. In *MTNS2004, Sixteenth International Symposium on Mathematical Theory of Networks and Systems, Katholieke Universiteit Leuven, Belgium*, pages 1–6, 2004.

[16] R.V. Sabariego, J Gyselinck, P. Dular, J. De Coster, F Henrotte, and K. Hameyer. Coupled mechanicalelectrostatic febe analysis with fmm acceleration: Application to a shunt capacitive mems switch. *COMPEL - The international journal for computation and mathematics in electrical and electronic engineering,*, 23(4):876–884, 2004.

[17] W. H. Schilders, H.A. van der Vorst, and J. (Eds.) Rommes. *Model Order Reduction: Theory, Research Aspects and Applications.* Springer, 2008.

[18] G. Schrag and G Wachutka. Systemlevel modeling of MEMS using generalized kirchhoffian networks basic principles. In *System-level Modeling of MEMS (T.Bechtold, G.Schrag, L.Feng (Eds))*, pages 19–51. Wiley-VCH Verlag, 2013.